APRIL/MAY 2023

DMA22 — REAL ANALYSIS – II

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State Riemann Lebesgue lemma.

2. Define Fourier series.

3. Define total derivative.

4. Write Jacobian matrix.

5. Define implicit function.

6. What is extremum problem?

7. Define measurable set,

8. Define Lebesgue integral

9. Write the properties of Lebesgue integral.

10. Define metric space.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Assume that $f \in L(I)$, prove that for each real β, $\lim \int_I f(t)\sin(\alpha t + \beta)\,dt = 0$.

Or

(b) State and prove Riesz — Fischer theorem.

12. (a) State and prove mean value theorem.

Or

(b) If f is differentiable at C, prove that f is continuous at C.

13. (a) State and prove implicit function theorem.

Or

(b) If $f = u + iv$ is a complex valued function with a derivative at a point z in c, prove that $\int_f (z) = \left|f'(z)^2\right|$.

14. (a) If $G_1, G_2 \cdots$ are open subsets of $[a, b]$ prove that $\left|\bigcup_{n=1}^{\infty} G_n\right| \le \sum_{n=1}^{\infty} |G_n|$.

Or

(b) If f and g are measurable function on $[a, b]$ prove that $f + g, f - g, fg$ are also measurable functions. If $g(x) \neq 0 (a \le x \le b)$, prove that f / g is also measurable.

15. (a) If G_1 and G_2 are open sets of $[a, b]$ prove that $|G_1| + |G_2| = |G_1 \cup G_2| + |G_1 \cap G_2|$

Or

(b) State and prove Minkowski inequality.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If g is of bound variation on $[0, \delta]$, show that $\lim_{n \to +\infty} \frac{2}{\pi} \int_0^{\delta} g(t) \frac{(\sin \alpha t)}{t}\,dt = g(0+)$.

17. State and Prove Taylor's theorem.

18. Show that the subset E of $[a, b]$ is measurable if and only if give $e > 0$ there exists open sets G_1 and G_2 such that $G_1 \supset E, G_2 \supset E'$ and $|G_1 \cap G_2| < \in$.

19. If f is measurable function on $[a, b]$ and if $c \in R$, prove that $f + c, cf$ are also measurable.

20. State and prove Lebesgue Dominated convergence theorem.